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Pulse dispersion in multimode optical fibres with alternate thin and thick layers

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Abstract : The present work is a theoretical study of the effect of introducing a thin layer of intermediate refractive index between every two consecutive layers in an equal-width multilayered optical fibre. This provides a simple but realistic model of an equal-width multilayered fibre in which errors in refractive index and layer-width are brought in during fabrication process. It has been found that there is a very small deterioration in the dispersion performance when the sandwitched layers are of small relative width (5% and 10%). When the intermediate layer is wider (say, 20% of the broad layer width), it can no longer be considered as an error but a change in design, and not unexpectedly the dispersion performance of the fibre improves, for in this case the fibre approaches a graded-index design. The main features of this analysis are independent of absolute linear dimensions.

Keywords : Optical fibres, multimode fibres, pulse dispersion.

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1. Introduction

Considerable work has been done on pulse broadening in multimode fibres (Goyal and Karstensen 1982, Kitayama and Seikài 1982) in the last few years. In particular, multilayer-multimode fibres have been considered for reduced pulse dispersion in comparison to that in step-index fibres. Petrovic (1980) using the ray-optic technique, predicted that fibres with only a small number of layers can reduce pulse broadening fairly well. The use of the ray-theoretical analysis is justified because the suitability of the mode theory is limited to only a small number of modes. It was also reported that the dispersion is a sensitive function of the index profile.

The present authors have recently reported (Subramaniyam *et al* 1988a, b) a comparative study of pulse-broadening factors (root-mean-square pulse widths) in multimode optical fibres with different types of layer-width distributions. In that paper, the parabolic graded-index distribution was simulated in multilayered fibres by choosing the Gaussian curve-fitting method. In another recent communication, the authors have reported (Subramaniyam *et al* 1988a, b) a ray-theoretic study of multilayered fibres with a composite α-profile. In Petrovic's work (Petrovic 1980), the refractive index changes from one layer to the adjacent layer abruptly at the boundary. We have a more realistic situation when the refractive index changes continuously but, rapidly over a small distance between two adjacent layers. One may say that a thin layer with an average refractive index $(n_i + n_{i+1})/2$ has been sandwitched between the layers of refractive indices n_i and n_{i+1} where 'i' is the layer number. The analysis of a fibre with this kind of layer-structure is important on two counts :

(i) It provides a basis for a tolerance theory for layer fabrication, and

(ii) It is worth studying what effect such a distribution may bring on the pulse-broadening.

Since no technique has yet been developed for fabricating multilayered fibres with great precision, the present day available multilayered fibres will have stretches of unintentional thin transition layers between successive thicker layers. However, a complete discussion of the situation will involve a statistical analysis, to take into account the randomness. Here the authors have taken a simpler systematic distribution of the layers in order to gain an insight into the problem. The present paper deals with the investigation of this kind of multilayered fibres in respect of pulse-broadening.

2. Theory

Figure 1 shows the axial section of a multilayered fibre with four thick layers of equal widths and three thin layers of common width spaced alternately. The refractive index of a thin layer is the arithmetic mean of those of the two thick layers surrounding it.

The refractive indices of the thick layers (as one goes from the axis to the cladding) are n_i ($i=1, 2, 3, 4$) and those of the sandwitched thin layers are $(n_i + n_{i+1})/2$. For the cladding, the refractive index is n_s. The condition for the confinement of the meridional rays to the core layers is given by

$$\cos\theta_1 \geqslant \cos\theta_{1c} = ! \tag{1}$$

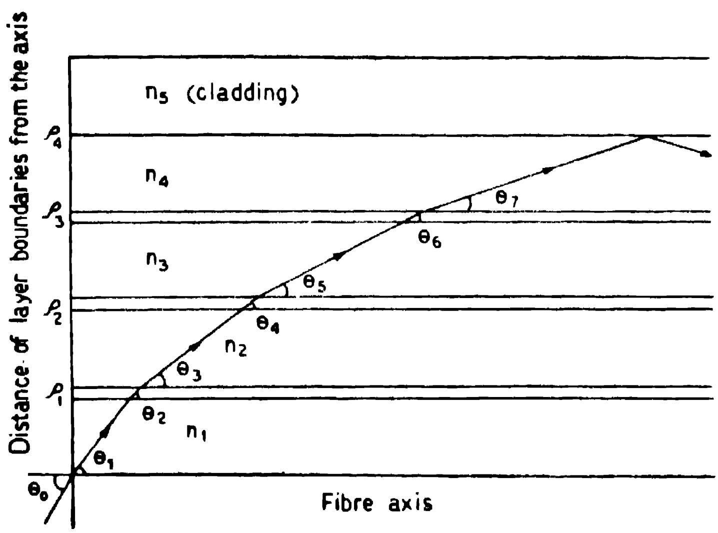

Figure 1. A multilayered optical fibre with four thick layers and three thin layers.

θ_1 being the angle between the ray and the fibre axis, and θ_{1c}, the critical angle corresponding to n_5 and n_1. Only the meridional rays have been considered for simplicity of analysis. The geometrical analysis of skew rays presents formidable difficulties and has not been dealt with here. We choose a profile function for the core in the form

$$n_i = n_1\left[1 - \Delta\left(\frac{i-1}{N}\right)^{\alpha}\right] \tag{2}$$

where α defines the profile ; $\Delta = n_5 - n_1$ is the index difference between the innermost core and the cladding ; n_i is the refractive index of the i-th layer and $n_1 = 1.5$.

As is well-known, the root-mean-square pulse width is given below

$$\sigma^2 = \frac{1}{P}\int_{-\infty}^{+\infty} h(t)t^2dt - \tau^2 \tag{3}$$

where

$$\tau = \frac{1}{P}\int_{-\infty}^{+\infty} h(t)tdt \tag{4}$$

is the pulse delay and $h(t)$ is the impulse response function. The pulse energy is given by

$$P = \int_{-\infty}^{+\infty} h(t)dt. \tag{5}$$

For a ray, propagating from the centre of the core through the layers to the boundary between the i^{th} layer and the $(i+1)^{th}$ layer, and then totally reflected at this boundary and propagating back to the axis, the ray transit time can be written as

$$t_i = \frac{\sum_1^i d_i \frac{n_i}{n_1} \operatorname{cosec} \theta_i}{\sum_1^i d_i \cot \theta_i} \cdot t_0 \tag{6}$$

where d_i is the thickness of the i-th layer for $i \neq 1$ and d_1 is the radius of the innermost layer and

$$t_0 = \frac{n_1 z}{c} \tag{7}$$

is the propagation time of the ray along the fibre axis, z is the path length along the axis ; c is the free-space speed of light ; and the angles θ_i are related to θ_1 according to the relation,

$$\cos \theta_i = \frac{n_1}{n_i} \cos \theta_1, \ (i=2, 3, 4) \tag{8}$$

Let us introduce the t_i's as obtained from eq. (6) into the expressions for the root-mean-square pulse width σ, and the pulse delay τ as expressed in eqs. (3, 4). We thus obtain

$$\tau = \frac{1}{\int_0^{\theta_{10}} h(\theta_1) d\theta_1} \sum_{i=1}^{N} \int_{\beta_i}^{\beta_{i+1}} t_i(\theta_1) h(\theta_1) d\theta_1 \tag{9}$$

$$\sigma^2 = \frac{1}{\int_0^{\theta_{10}} h(\theta_1) d\theta_1} \sum_{i=1}^{N} \int_{\beta_i}^{\beta_{i+1}} t_i^2(\theta_1) h(\theta_1) d\theta_1 - \tau^2 \tag{10}$$

where N is the number of core layers

and

$$\cos \beta_i = n_i / n_1 \tag{11}$$

The function $h(\theta_1)$ can be calculated from the relation

$$h(\theta_0) = h_m \cos \theta_0 \sin \theta_0 \tag{12}$$

where

$$n_0 \sin \theta_0 = n_1 \sin \theta_1 \tag{13}$$

h_m being the height of the input impulse (Snyder 1983).

If, in the fabricating process, the refractive index while changing from one thick layer to the next, varies over a thin layer whose width is a certain percentage of the common thick layer-width, the situation can be viewed as an error introduced during the course of fabrication of an equal width multilayered fibre. Thus one can estimate the effect of average unintentional refractive-index variation from one layer to the next, on root-mean-square (r. m. s.) pulse-broadening. Again, such changes in refractive indices which are deliberate may bring about a significant change in the root-mean-square pulse width. Assuming that the thin layer width is 5%, 10% and 20% of the thick layer width, one can use eqs. (9) and (10) to calculate the ratio of the pulse broadening for a fibre with seven core layers (including all the alternate thick and thin layers) and that for a fibre with a single core layer. This gives σ_7/σ_1 as a function of α. The widths of layers are derived from the following equation

$$\rho = 4a + 3x \tag{14}$$

where a is the common width of the thick layers, x the common width of the thin layers, and ρ is the total radius of the core. Simplifying eq. (14), we get

$$\frac{x}{a} = \frac{1}{3}\left[\frac{\rho}{a} - 4\right]. \tag{15}$$

This ratio can be expressed in percentage as

$$p = \frac{100}{3}\left[\frac{\rho}{a} - 4\right]. \tag{16}$$

Using eqs. (15) and (16), the widths of layers are given by

$$a = \frac{100\rho}{3p + 400} \tag{17}$$

$$x = \frac{p\rho}{3p + 400} \tag{18}$$

3. Computations

Various σ ratios have been computed using eqs. (9) and (10) for two values of Δ and for different layer-width distributions. All the computations are made using ICL 1904S. These results have been shown in Tables 1 and 2 and also graphically in Figure 2. It may be observed that the ratios for $\Delta = 0.01$ and for $\Delta = 0.02$ show differences only at the third decimal place. Therefore, their graphical representations in Figure 2 have merged together.

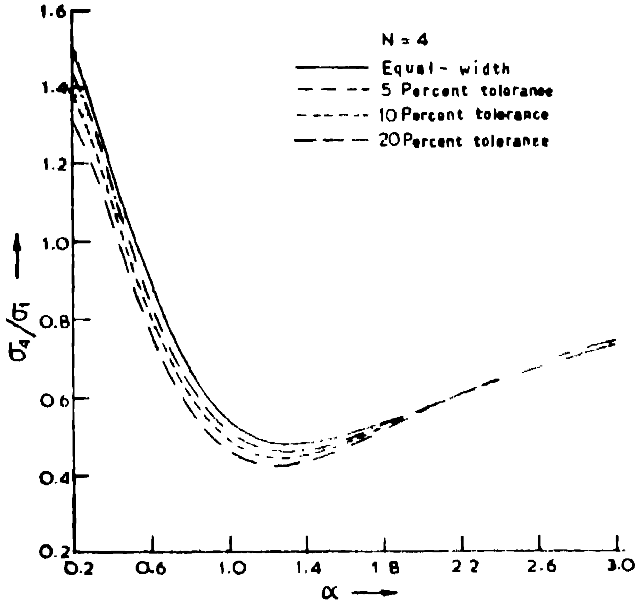

Figure 2. Pulse-broadening ratios as functions of α for equal-width and unequal-width fibres.

Table I. Ratios of pulse-broadening.

α	$\Delta=0.01$ σ_7/σ_1 with 5% tolerance	$\Delta=0.01$ σ_7/σ_1 with 10% tolerance	$\Delta=0.01$ σ_7/σ_1 with 20% tolerance	$\Delta=0.01$ Equal-width σ_4/σ_1
0.2	1.429	1.379	1.306	1.498
0.4	1.132	1.094	1.038	1.182
0.6	0.830	0.801	0.759	0.869
0.8	0.618	0.595	0.561	0.650
1.0	0.499	0.480	0.452	0.526
1.2	0.455	0.439	0.418	0.476
1.4	0.457	0.446	0.432	0.473
1.6	0.483	0.476	0.467	0.494
1.8	0.519	0.514	0.511	0.526
2.0	0.557	0.556	0.555	0.561
2.2	0.596	0.596	0.598	0.598
2.4	0.633	0.635	0.639	0.633
2.6	0.668	0.670	0.676	0.666
2.8	0.701	0.704	0.710	0.698
3.0	0.731	0.734	0.741	0.727

Table 2. Ratios of pulse-broadening.

α	$\Delta=0.02$ σ_7/σ_1 with 5% tolerance	$\Delta=0.02$ σ_7/σ_1 with 10% tolerance	$\Delta=0.02$ σ_7/σ_1 with 20% tolerance
0.2	1.416	1.365	1.293
0.4	1.121	1.083	1.028
0.6	0.822	0.793	0.751
0.8	0.612	0.588	0.555
1.0	0.494	0.475	0.448
1.2	0.451	0.436	0.415
1.4	0.454	0.443	0.430
1.6	0.481	0.474	0.466
1.8	0.517	0.513	0.509
2.0	0.556	0.554	0.554
2.2	0.595	0.595	0.597
2.4	0.632	0.633	0.638
2.6	0.667	0.669	0.675
2.8	0.700	0.703	0.709
3.0	0.730	0.733	0.740

4. Theoretical results and discussion

As stated earlier, the curves in Figure 2 show computational results in respect of pulse-broadening for both equal-width and unequal-width multilayered fibres. We find that the layer-width distribution discussed above shows lesser pulse-broadening than that for equal-width distribution for $\alpha \leqslant 2.0$. Beyond this value of $\alpha(=2.0)$, all the curve merge with the equal-width distribution curve. We must note, however, that the curve corresponding to a fibre in which equal-width layers have, between them, narrower layers of 20% width, shows a slight deviation from the above general tendency. In particular, it shows the lowest pulse dispersion for $\alpha \leqslant 2.0$, but a slightly greater pulse dispersion than those of the other curves beyond $\alpha = 2.2$. A minimum value of 0.418 for the ratio σ_7/σ_1 is obtained at $\alpha = 1.2$. Therefore, we can conclude that the introduction of thin layers only makes a slight change in (rather, an improvement on) the dispersion performance of multilayered fibres and the profile approaches a graded-index design for $\alpha = 2.0$.

The reason for the slight improvement of dispersion performance, contrary to one's expectation, must be sought in the systematic introduction of the narrow layers between the thicker layers. If randomness were introduced in the values of the refractive indices and in the positions of the narrow layers, the dispersion performance would have deteriorated. The most cautious conclusion that one can draw from the study of the above simple model is that fabrication errors in the profile will introduce small changes in dispersion performance particularly for values of $\alpha \geqslant 2.0$. For smaller values of α (such as a triangular profile $\alpha = 1$) fabrication errors may even improve the performance slightly. It may also be noticed that the main features of the above analysis are independent of absolute linear dimensions. However, since the analysis is geometrical, it can not be extended to the wave picture where the operating wavelength becomes important and fibres with very small core diameters may behave as single mode fibres or few mode fibres.

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